

Project - Modeling the Motion of a Spring

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*Please read the entire project before doing any work. Due Date: _____

30/50

Consider a weight attached to a spring that is suspended from a horizontal bar as illustrated in the figure. When the object comes to rest we say it is at "equilibrium" which is labeled 0 on the vertical number line. If you give the weight a push, either up or down, it will start to move and the motion can be modeled using sine and cosine functions. The "stiffness" of the spring and the mass of the object affect how far the object moves from the equilibrium position. The initial velocity and initial position also affect the motion of the spring. (We don't always start at the point of equilibrium.)

If we neglect any damping forces (air resistance etc.) then the motion of the spring can be modeled by:

$$x(t) = \frac{v_0}{\omega} \sin(\omega t) + x_0 \cos(\omega t)$$

where $x(t)$ is the position of the object along the number line at time t . The other quantities are constants: ω (the Greek letter "omega") is a constant that depends on the stiffness of the spring and the mass of the weight, v_0 is the initial velocity, and x_0 is the initial position of the object.

Model the motion of a weight on a spring:

Suppose a weight is set in motion from a position 3 centimeters below the equilibrium position and with a downward velocity of 4 centimeters per second. (Please note that the vertical number line used for position is "upside down". This is a convention from physics and it means that positions below equilibrium actually correspond to a positive value.) Assume that the spring stiffness and mass of the weight mean that $\omega = 2$ for this system.

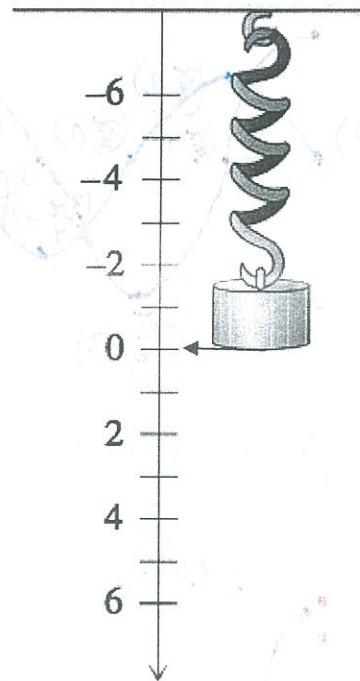
Part I

1) Write the function $x(t)$ that gives the position of the weight as a function of time t in seconds. (Your function should consist of a sine term and a cosine term.)

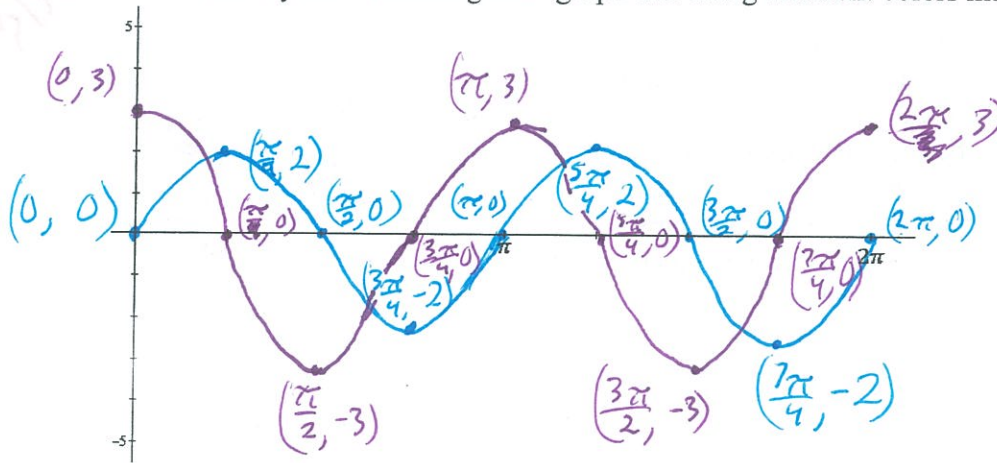
$$x(t) = \frac{4}{2} \sin(2t) + 3 \cos(2t)$$

↑
Simplify

$$x(t) = 2 \sin(2t) + 3 \cos(2t)$$



2) Graph the separate sine and cosine components of your function from question 1) on the same set of axes. That is, graph $x_1 = \frac{v_0}{\omega} \sin(\omega t)$ and $x_2 = x_0 \cos(\omega t)$ on the set of axes below. (Sketch these graphs by hand and show two full cycles. Labeling each graph and using different colors may be helpful.)



$$x_1 = 2 \sin(2t)$$

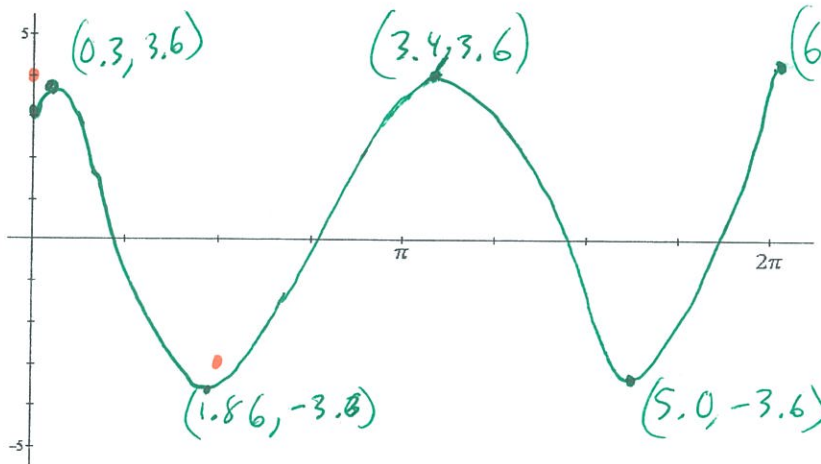
P: π Amp: 2

$$x_2 = 3 \cos(2t)$$

P: π Amp: 3

3) Use your calculator to graph the entire function from question 1). Use the window settings indicated below. Sketch what you see on your calculator display.

xmin = 0
 xmax = 2π
 xscl = $\frac{\pi}{4}$
 ymin = -4
 ymax = 4
 yscl = 1



$$x = 2 \sin(2t) + 3 \cos(2t)$$

P: π Amp: 3.6

4) Write an equation for the graph in question 3 in the form $x(t) = A \cos[B(t - C)]$. (Use your trace or maximum feature to help you find approximate values for A, B, and C. Don't worry, there may be some decimals in your answer.)

$$x(t) = 3.6 [\cos(2(t - 0.267))]]$$

5) How are the graphs from question 2 related to the graph in question 3? Are the values for the period and amplitude the same or different? Why do you think we see these results? Please write a one or two sentence explanation.

The period is the same because the period change factor is the same in all of the graphs. However, the amplitude for all of the graphs are different because in the first graph they were graphed separately whereas in the second it is the combination of the first graphs.

Part II:

6) Prove that the following is an identity where A is a positive constant. (Hint: Recall the sum and difference identities.)

$$A \sin(\phi) \sin(\omega t) + A \cos(\phi) \cos(\omega t) = A \cos(\omega t - \phi)$$

$A \sin(\phi) \sin(\omega t) + A \cos(\phi) \cos(\omega t)$	$= A(\sin(\phi) \sin(\omega t) + \cos(\phi) \cos(\omega t))$	Factor out A
	$= A(\cos(\omega t) \cos(\phi) + \sin(\omega t) \sin(\phi))$	Commutative Property of Addition
	$= A[\cos(\omega t) \cos(\phi) + \sin(\omega t) \sin(\phi)]$	Commutative Property of Multiplication
	$= A \cos(\omega t - \phi)$	Difference Identity for cosine

Part III:

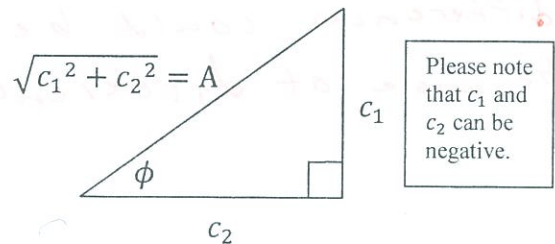
Use the following definitions and your result from part II to rewrite an expression of the form

$$\underbrace{c_1 \sin(\omega t) + c_2 \cos(\omega t)}_{[1]} = \underbrace{A \sin(\phi) \sin(\omega t) + A \cos(\phi) \cos(\omega t)}_{[2]} \text{ in terms of a single cosine function}$$

such as $A \cos(\omega t - \phi)$ where:

$$A = \sqrt{c_1^2 + c_2^2}$$

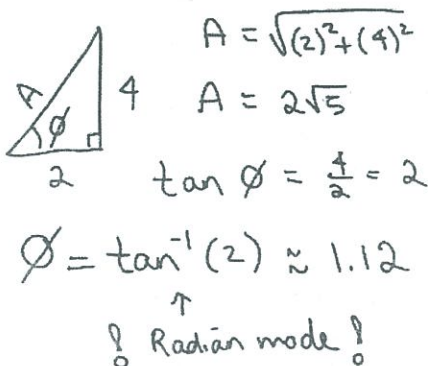
$$\left. \begin{array}{l} [1] \ A \sin(\phi) = c_1 \Rightarrow \sin(\phi) = \frac{c_1}{A} \\ [2] \ A \cos(\phi) = c_2 \Rightarrow \cos(\phi) = \frac{c_2}{A} \end{array} \right\} \tan(\phi) = \frac{c_1}{c_2}$$



The formulas above will help you find A (the amplitude of the cosine function) and ϕ (the Greek letter "phi" [rhymes with "lie"] and is called the phase angle measured in radians).

Consider the following example which is completed with the definitions and identities above:

Example- Write $4 \sin 3t + 2 \cos 3t$ in terms of a cosine function.



$$4 \sin 3t + 2 \cos 3t \approx 2\sqrt{5} \cos(3t - 1.12)$$

$$\approx 2\sqrt{5} \cos[3(t - 0.37)]$$

7) Rewrite your function from question 1) in terms of a cosine function of the form $y = A \cos(\omega t - \phi)$. Show your work.

$$2 \sin(2t) + 3 \cos(2t) \approx \sqrt{13} [\cos(2t - 0.588)]$$

$$\text{Amp: } \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13}$$

$$\boxed{\sqrt{13} [\cos(2(t - 0.29))]}$$

$$\text{PS: } \tan^{-1}\left(\frac{2}{3}\right) = 0.588$$

8) Compare your function from questions 4) and 7). What do you observe? Write a one or two sentence explanation.

The two equations are nearly equal. The little differences could be due to a lack of exactness or because of different rounding.

ePortfolio Posting:

All Math 1060 students must post a copy of the mass-spring project in their General Education ePortfolio. The completed project should be scanned and posted in a PDF format. The ePortfolio must be linked to your MyPage. Students must include some reflective writing about the project in the ePortfolio. The reflection should include a response to the following:

Give some examples of mass-spring systems that are around us in everyday life. Describe why it would be important to understand the amplitude, period, and frequency of these systems. Did the project change the way you think about how trigonometry and math can be applied to the real world? State some of your thoughts that changed and why. If this project did not change the way you think, write how this project gave further evidence to support your existing opinion about applying trigonometry and math in general. Be specific.

ePortfolio Posting and Reflection will be graded as follows:

3 Points: Project done with only minor mistakes, is posted in the ePortfolio, and the ePortfolio is linked to MyPage by the deadline. The reflective writing addresses at least one example of a mass spring system and the other questions posed above. The reflection is at least two paragraphs in length.

7 Points: Project done with no more than one minor mistake, is posted in the ePortfolio, and the ePortfolio is linked to MyPage by the deadline. The reflective writing addresses 2 or more examples of mass spring systems and the other questions posed above. The reflection is well written. The work should be clear, well organized and at least one page in length.

10 Points: (maximum- only awarded for truly outstanding work): Project is done with no mistakes, is posted in the ePortfolio, and the ePortfolio is linked to MyPage by the deadline. The reflective writing addresses 2 or more examples of mass spring systems and the other questions posed above. The reflective writing is exemplary. The work offers unique observations and draws insightful, carefully qualified conclusions and is at least one page in length.